Summary of Tangents, Linearizations, and Differentials in Various Dimensions

In two dimensions, a linear equation is $Ax + By = C$. Its graph is a line. If the line is nonvertical (i.e., if B is nonzero), it may be written is point-slope form, $y = m(x - x_0) + y_0$.

In three dimensions, a linear equation is $Ax + By + Cz = D$. Its graph is a plane. If the plane is nonvertical (i.e., if C is nonzero), it may be written is point-slope form,
$z = m_1(x - x_0) + m_2(y - y_0) + z_0$.

In four dimensions, a linear equation is $Ax + By + Cz + Dw = E$. Its graph is a hyper-plane. If the hyper-plane is nonvertical (i.e., if D is nonzero), it may be written is point-slope form,
$w = m_1(x - x_0) + m_2(y - y_0) + m_3(z - z_0) + w_0$.

A real-valued function with a one-dimensional domain, $y = f(x)$, has a two-dimensional graph (i.e., a curve). Let $y_0 = f(x_0)$. The tangent line at the point (x_0, y_0) is
$y = f'(x_0)(x - x_0) + y_0$.

A real-valued function with a two-dimensional domain, $z = f(x,y)$, has a three-dimensional graph (i.e., a surface). Let $z_0 = f(x_0, y_0)$. The tangent plane at the point (x_0, y_0, z_0) is
$z = f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0) + z_0$.

A real-valued function with a three-dimensional domain, $w = f(x,y,z)$, has a four-dimensional graph (i.e., a hyper-surface). Let $w_0 = f(x_0, y_0, z_0)$. The tangent hyper-plane at the point (x_0, y_0, z_0, w_0) is
$w = f_x(x_0,y_0,z_0)(x - x_0) + f_y(x_0,y_0,z_0)(y - y_0) + f_z(x_0,y_0,z_0)(z - z_0) + w_0$.

For $y = f(x)$, the linearization at x_0 is $L(x) = f'(x_0)(x - x_0) + y_0$.
$\Delta f \approx \Delta L = df = dy = f'(x_0)dx$.

For $z = f(x,y)$, the linearization at (x_0, y_0) is $L(x,y) = f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0) + z_0$.
$\Delta f \approx \Delta L = df = dz = f_x(x_0,y_0)dx + f_y(x_0,y_0)dy = \nabla f(x_0,y_0) \cdot < dx, dy >$

For $w = f(x,y,z)$, the linearization at (x_0, y_0, z_0) is
$L(x,y,z) = f_x(x_0,y_0,z_0)(x - x_0) + f_y(x_0,y_0,z_0)(y - y_0) + f_z(x_0,y_0,z_0)(z - z_0) + w_0$.
$\Delta f \approx \Delta L = df = dw = f_x(x_0,y_0,z_0)dx + f_y(x_0,y_0,z_0)dy + f_z(x_0,y_0,z_0)dz =$
$\nabla f(x_0,y_0,z_0) \cdot < dx, dy, dz >$